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Minimum solid area models applied to the prediction of Young's modulus for cancellous bone

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In developing models for the mechanical behavior of cancellous bone, accurate prediction of Young's modulus as a function of the pore fraction and morphology is a requirement. Previous workers have suggested models which provide good statistical fits, but most of these models are highly idealized, with no treatment of the actual morphology of the porosity. In the field of engineering ceramics, simple minimum solid area models have been developed over the past four decades to describe the mechanical properties of porous structural ceramics. This paper applies these models to data for cancellous bone, and it is shown that one, developed specifically for high porosity materials, gives realistic predictions of tissue modulus and a good statistical fit to well-established data. This model should prove to be useful in biomechanical analyses involving cancellous bone tissue.

1. Introduction

Young's modulus is a fundamental property of materials. In biomechanics, knowledge of the Young modulus of cancellous bone, and its variation with porosity, is required for, amongst other things, finite element analysis of orthopaedic implant systems.

Cancellous bone presents particular difficulties when attempting to produce a model for Young's modulus. At the smallest scale, the individual trabeculae are complex fibrous composites of inorganic bone mineral interwoven with a largely organic phase, which is itself a composite of mineral particles in a collagen matrix (we shall refer to the trabecular-level composite as tissue). At the next level up, the trabecular tissue forms rods and plates which enclose large voids (macropores), which may be roughly spherical (i.e., equiaxed, in isotropic cancellous bone) or elongated (anisotropic bone); these structures are illustrated in Figs. 1 and 2.

Much work has been carried out on modeling the Young modulus of bone at a macroscopic level, based on curve-fitting using empirical formulae, usually with the apparent density (which is a function of the overall porosity level) as the independent variable, e.g. [1–4]. Perhaps the best-known of these models is that of Carter and Hayes, who proposed that the Young modulus is proportional to the cube of the apparent density (ρ_{app}) of the bone, with a correction for strain rate.

While these empirical models provide good fits to specific sets of data, they contain no explicit treatment of the pore size, shape, and orientation, or of the geometry of the solid structures which define the pores. As a consequence, in many cases the equations cannot be applied to other data sets.

Gibson [5] and Rajan [6] re-analyzed the data of Carter and Hayes [1] for cancellous bone, using analytical models developed for cellular solids by Gibson and Ashby [7], and showed that while $E \propto \rho^3$ for both asymmetric and equiaxed closed porosity, a better relationship for open porosity was $E \propto \rho^2$.

A rigorous multiple-regression statistical analysis of many sets of data for the mechanical properties of cancellous bone, isolating the effects of factors such as orientation of the bone sample and the type of test method, was carried out by Rice *et al*. [8]. The complete model included terms for proportionality to ρ, ρ^2, and ρ^3 (the linear relationship deriving from the work of Christensen [9], and the quadratic and cubic relationships in accordance with the models of Gibson and Ashby [7]), and it was concluded that Young's modulus is proportional to ρ^2 with $p < 0.001$.

Finite element analysis (FEA) has also been used to determine the tissue modulus (E_0, the Young modulus of the trabecular tissue) of cancellous bone: van Rietbergen *et al*. [10] created a model based on a digitized $7 \times 7 \times 7$ mm cube of cancellous bone. A range of values for E_0 was then determined by sophisticated and computationally intensive FEA, in which Young's modulus for the overall block was matched to values from empirical equations proposed by Hodgkinson and Currey [3]. This is in contrast to the approach of Carter

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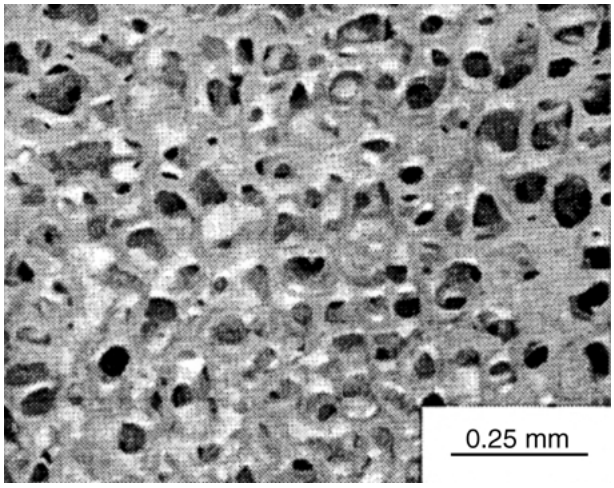

Figure 1 Morphology typical of cancellous bone with equiaxed pores (isotropic).

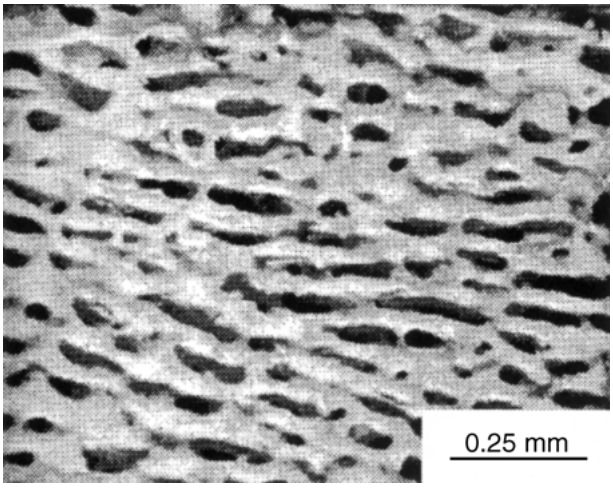

Figure 2 Morphology typical of cancellous bone with elongated pores (anisotropic).

and Hayes [1], who followed Wolff [11] in assuming that dense trabecular tissue has elastic properties similar to dense cortical bone (other workers have claimed that this is an incorrect assumption [8, 12], although recent nanoindentation studies on trabeculae by Rho *et al*. [13] have produced values for E_0 similar to cortical bone).

Effective methods of predicting the elastic moduli of engineering materials have received much attention over the last four decades, driven, in the case of refractories and advanced structural ceramics, by the practical need to quantify effects of processing and defects on the bulk mechanical properties. There are three main approaches: composite theory [14–17], cellular solids [7], and minimum solid area [18–24]. The first approach assumes a two-phase material, with one phase having zero stiffness; the latter two assume a single phase permeated with voids.

Although cancellous bone is undoubtedly a composite material at the tissue level, the open macrostructure defined by the trabeculae may be treated as a porous, single-phase material, with homogenized properties. Piekarski [25] has already suggested the application, to bone, of an equation developed by Mackenzie [26], and used by Coble and Kingery [27] to describe alumina ceramics containing between 5% and 50% artificially introduced isolated porosity; however, this model has been replaced for ceramics work by those of Spriggs [18], Rice [22], and Nielsen [16] (described below), and will not be considered here.

This paper considers the applicability of minimum solid area models to the prediction of Young's modulus for cancellous bone. These have the advantage over the models currently used in biomechanics in that they incorporate a parameter describing pore morphology and packing, as well as pore volume fraction. Non-linear regression analyses of the minimum solid area and other models (such as cellular solids) were carried out in the present work using the well-established data of Carter and Hayes [1]. These analyses yielded statistical information, allowing comparison of the different models.

2. Theory and models

2.1. Models currently in use for cancellous bone

Carter and Hayes [1] carried out compression tests on blocks of cancellous bone both with and without marrow present, in order to measure the Young modulus. They used five strain rates ($\dot{\varepsilon} = 0.001$, 0.01, 0.1, 1.0, and $10.0\,s^{-1}$), in order to study the sensitivity of properties to strain rate. Porous platens were used so that the marrow would not be constrained, allowing it to flow out of the bone blocks; however, at the highest strain rate the porosity in the platens was too constricting, resulting in the marrow becoming pressurized within the cancellous bone pore system (accordingly, their $\dot{\varepsilon} = 10\,s^{-1}$ data for specimens containing marrow are not used in the current work). Carter and Hayes fitted straight lines through log–log plots of their data (not strictly a valid approach, see Myers [28]) and found that the Young modulus (i) was proportional to the cube of the apparent density (ρ_{app}) and (ii) exhibited a slight strain-rate sensitivity, proportional to $\dot{\varepsilon}^{0.06}$. Thus, the expression proposed by Carter and Hayes for the variation of Young's modulus of cancellous bone with apparent density and strain-rate is:

$$E = k_{CH}\dot{\varepsilon}^{0.06}\rho_{app}^3 \tag{1}$$

where k_{CH} is a constant relating to the tissue modulus ($k_{CH} \approx E_0/\rho_0^3$; E_0 and ρ_0 are the Young modulus and density of dense trabecular tissue, respectively), $\dot{\varepsilon}$ is the strain rate, and ρ_{app} is the apparent density of the block of bone.

Gibson [5] and Rajan [6] fitted log–log models, derived by Gibson and Ashby [7] for cellular materials, to the data of Carter and Hayes [1]. The two models are:

$$E = k_{GA2}\rho_{app}^2 \tag{2}$$

$$E = k_{GA3}\rho_{app}^3 \tag{3}$$

where k_{GA2} and k_{GA3} are constants relating to the tissue modulus ($k_{GA2} = E_0/\rho_0^2$ and $k_{GA3} = E_0/\rho_0^3$).

The statistical analysis of Rice *et al*. [8] showed that Equation 2 was not substantially worse at predicting Young's modulus than a linear combination of Equations 2 and 3, and that either of these was somewhat better than Equation 3 on its own.

2.2. Models currently in use for ceramics

Spriggs [18] first proposed that the earlier work of Ryshkewitch [29], Duckworth [30], and Knudsen [31] – who used minimum solid area models to predict the influence of porosity on strength of ceramics – could be extended to the elastic moduli (accordingly, we shall refer to this as the Spriggs model, following Wachtman [32]). The Spriggs equation is a simple exponential degradation of the modulus of the fully dense material, and has the form:

$$E = E_0 \exp(-bP) \tag{4}$$

where E is the Young modulus of the porous material, b is a pore-geometry term relating to the pores defined by the packing of the solid phase, and P is the pore volume fraction (pore volume/total volume). Spriggs empirically related b to the porosity (pore volume fraction, P) introduced by the fabrication method for the ceramic. Brown *et al*. [33] presented analytically derived values of b for different pore geometries and orientations, but only in the context of strength. For elastic moduli, Rice [24] identified a similar range of empirically derived values for b relating to pore geometry. The Spriggs model ceases to apply at high porosities, with $E \to E_0 \exp(-b)$, rather than $E \to 0$, as $P \to 1$.

Rice [22] modified the Spriggs model to apply at high (but not low) porosities by interchanging the solid and pore phases (i.e., considering the packing of the pores rather than the particles), and modifying the pore geometry term accordingly, giving:

$$E = E_0[1 - \exp\{-b'(1-P)\}] \tag{5}$$

where b' is a geometry term relating to the solid phase, as defined by the packing and shape of the pores.

Another model, initially deriving from work on civil engineering composites such as concrete, is that of Nielsen [15, 16]. This has the form:

$$E = E_0 \frac{(1-P)^2}{1 + \{(1/k_N) - 1\}P} \tag{6}$$

where k_N is the pore-geometry factor ($0 < k_N \leq 1$).

2.3. Curve fitting

The models described above were applied here to experimental data published by Carter and Hayes [1] for the Young modulus of human cancellous bone, measured in compression. The data were extracted from the published graphs using the DataThief computer application (version 2.0; K. Huyser and J. van der Laan, National Institute for Nuclear Physics and High Energy Physics, Amsterdam, The Netherlands). Apparent density (ρ_{app}) was converted to volume fraction porosity (P):

$$P = 1 - (\rho_{app}/\rho_0) \tag{7}$$

assuming a trabecular tissue density, ρ_0, of 1.82 g/cm^3 [34].

All models to be fitted were non-linear in ρ_{app} or P, and although all models except for those of Rice and Nielsen could be log-transformed in order to linearize the problem, this was not done, as the structure of the errors would then be incorrect, and the parameters obtained would not be the same as those from non-linear fitting of the untransformed model [28].

Curve-fitting was carried out in Excel 4.0 for Macintosh (Microsoft Corporation, Redmond, WA, USA). The model parameters (such as E_0 and b in the Spriggs equation) were varied by the Solver add-in (which uses the generalized reduced gradient (GRG2) non-linear optimization algorithm) to minimize the sum of the squares of the differences (sum of the squares for error, SSE) between the experimental data (E_{obs}) and calculated values (E_{model}), for each model [35]:

$$\text{SSE} = \sum (E_{obs} - E_{model})^2 \tag{8}$$

Also calculated was SS_{yy}, the sum of the squares of the deviation from the mean ($\bar{E}_{obs}$):

$$SS_{yy} = \sum (E_{obs} - \bar{E}_{obs})^2 \tag{9}$$

This allowed the calculation of the coefficient of determination, r^2, for the fitted model:

$$r^2 = \frac{SS_{yy} - \text{SSE}}{SS_{yy}} \tag{10}$$

and from this a t-statistic could be derived:

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \tag{11}$$

(where n is the sample size) permitting computation of a significance level, p, for the fit.

3. Results

All of the Carter and Hayes [1] data for human cancellous bone, except for those measured at the highest strain-rate (10 s^{-1}) for specimens with marrow, were used here. Since Carter and Hayes stated that strain-rate had only a small effect on the measured modulus, the data were pooled (as they were by Gibson [5] and Rajan [6]), irrespective of strain rate, for fitting with the Gibson and Ashby [7], Spriggs [18], Rice [22], and Nielsen [16] models. When fitting with the model of Carter and Hayes [1] the same data were used, but account was taken of the strain rate, as per Equation 1. Three data sets were fitted: cancellous bone (i) with marrow, (ii) without marrow, and (iii) these two sets combined. The models were all fitted to the data sets using Excel, as described above, and the results are summarized in Table I.

During curve fitting, the Solver was allowed to vary the value of k_{CH} in the Carter and Hayes model – this was not constrained to the value of 3790 proposed by Carter and Hayes, as it was found to give an extremely poor fit. The regression line used by Carter and Hayes appears to have been drawn to minimize SSE in the log domain, rather than the linear domain, thus giving excessive weighting to samples with low apparent densities [28]. The Solver was allowed to vary k_{GA2} and k_{GA3} in the Gibson and Ashby models for the same reason. When fitting the Spriggs, Rice, and Nielsen models, the Solver was permitted to vary both E_0 and b, E_0 and b', and E_0 and k_N, respectively. Figs. 3–5 show the best-fit curves obtained for the Gibson and Ashby, Spriggs, and Rice models (note that, for consistency with previously

TABLE I Fitted parameters for the models of Carter and Hayes [1], Gibson and Ashby [7], Spriggs [18], Rice [22], and Nielsen [16], using data from Carter and Hayes [1]

Model	Parameters		r^2	p
Carter and Hayes	k_{CH}			
With marrow	1385		0.585	2×10^{-8}
Without marrow	2386		0.396	7×10^{-7}
Combined	1617		0.288	6×10^{-8}
Gibson and Ashby (ρ^2)	k_{GA2}	$\rightarrow E_0$(MPa)		
With marrow	517	1714	0.581	3×10^{-8}
Without marrow	811	2686	0.678	1×10^{-13}
Combined	601	1990	0.477	7×10^{-14}
Gibson and Ashby (ρ^3)	k_{GA3}	$\rightarrow E_0$(MPa)		
With marrow	1053	6348	0.373	5×10^{-5}
Without marrow	2128	12 828	0.381	1×10^{-6}
Combined	1255	7566	0.065	2×10^{-2}
Spriggs	b	E_0(MPa)		
With marrow	7.08	20 855	0.607	8×10^{-9}
Without marrow	8.99	133 456	0.688	5×10^{-14}
Combined	6.51	14 994	0.572	1×10^{-17}
Rice	b'	E_0(MPa)		
With marrow	0.046	8908	0.527	3×10^{-7}
Without marrow	0.072	6680	0.638	2×10^{-12}
Combined	0.089	4984	0.601	1×10^{-17}
Nielsen	k_N	E_0(MPa)		
With marrow	3.77	775	Invalid result ($k_N > 1$)	
Without marrow	5.89	884	Invalid result ($k_N > 1$)	
Combined	13.49	573	Invalid result ($k_N > 1$)	

published graphs, these are log–log plots, and that a fit which minimizes SSE in the linear domain can produce a regression line which optically appears to be poor in the log domain). The Nielsen model was not plotted, as in all cases it converged to a value for $k_N > 1$, i.e., outside the legitimate range.

4. Discussion

For the purposes of this comparison, a realistic value for the Young modulus of trabecular bone tissue (E_0) was considered to be one which lay in the range reported in the literature [36–39, 12, 13] (Table II, mean of 12.3 GPa). Using finite element methods, van Rietbergen *et al*. [10] estimated the tissue modulus of cancellous bone to lie in the range 2.23–10.1 GPa, with a mean of 5.91 GPa, which is slightly greater than the smallest value in Table II.

The Gibson and Ashby ρ^2 model was found to be a good fit to the Carter and Hayes data (Table I), in most cases superior to the Carter and Hayes model, despite the latter's additional correction for strain rate; this is also in agreement with the findings of Rice *et al*. [8]. However, the Gibson and Ashby ρ^2 model predicts (assuming a tissue density of 1.82 g/cm^3, and given that the model requires that $k_{GA2} = E_0/\rho_0^2$) Young moduli of 1.7 GPa (with marrow), 2.7 GPa (without marrow), and 2.0 GPa (combined data sets), all of which are low compared to the values in Table II and to those from the FE analysis of van Rietbergen *et al*. [10]. It would also have been expected that the model should predict slightly higher moduli for the samples containing marrow than for those without marrow as, despite the porous platens used by Carter and Hayes in their tests, some small pressurization of the pore fluid might be expected. However, this model predicted the reverse situation, with the marrow-free specimens appearing to be stiffer.

TABLE II Published values of E for cancellous bone

Source	E (GPa)
Townsend and Rose [36]	11.4
Ashman and Rho [37]	13.0
Ashman and Rho [37]	10.9
Mente and Lewis [38]	7.8
Choi *et al*. [39]	5.7
Rho *et al*. [12]	14.8
Rho *et al*. [13]	19.4
Rho *et al*. [13]	15.0
Mean of above	12.3
95% confidence limit	$\pm$ 3.6

In contrast, the Gibson and Ashby ρ^3 model (which, apart from the lack of strain-rate correction, is identical to the Carter and Hayes model) gave a poorer fit than all models except Nielsen's, but produced more realistic values for $E_0 (= k_{GA3}\rho_0^3)$ than the ρ^2 model: 6.3, 12.8, and 7.6 GPa, for the marrow, no marrow, and combined data sets, respectively. The poor fit accords well with the findings of the statistical study by Rice *et al*. [8]. Again, a higher modulus was predicted for the bone with the marrow removed than for the bone which contained marrow.

Despite excellent statistics, the Spriggs model predicted E_0 values of 20.8 GPa (with marrow), 133.5 GPa (without marrow), and 15.0 GPa (combined data). With the exception of the case of the bone without marrow, which has far too high a value, these figures for Young's modulus are in reasonable agreement with the range of experimental values in Table II. The pore geometry factor (b) lay within reasonable limits, except in the case of the bone with no marrow, where the Solver compensated for the high value for E_0 by setting b to an excessively large 8.99. Once again, the bone without marrow was predicted to be stiffer than bone containing

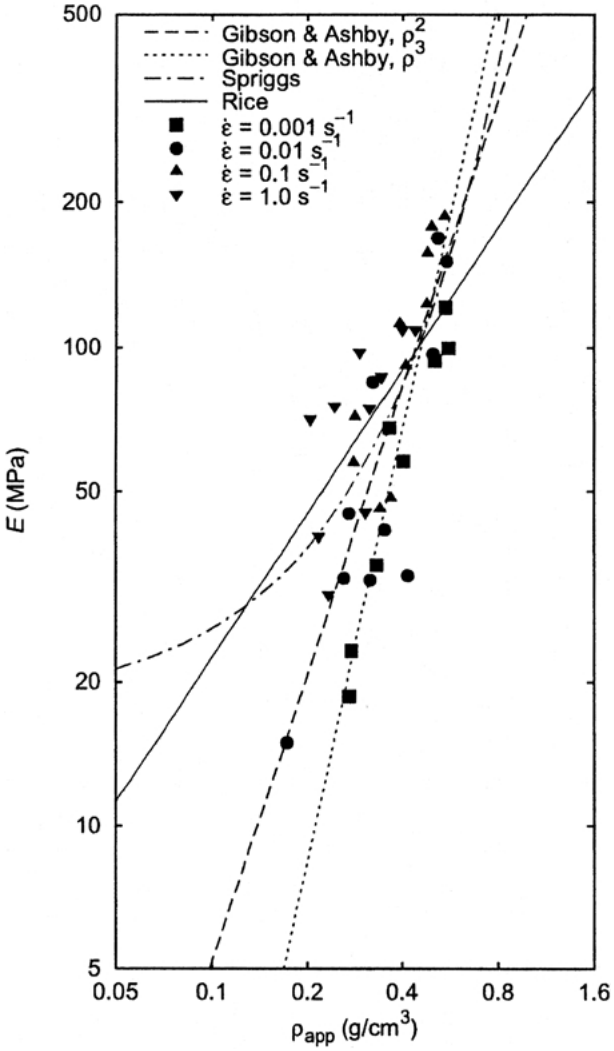

Figure 3 Young's modulus data from Carter and Hayes [1] for cancellous bone with marrow, showing best fit lines for the Gibson and Ashby, Spriggs, and Rice models.

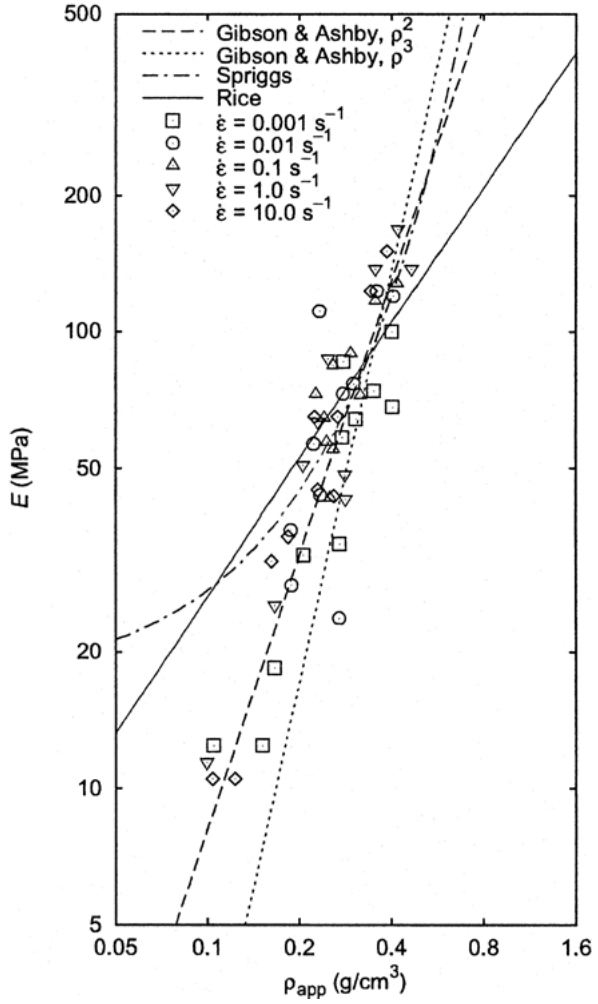

Figure 4 Young's modulus data from Carter and Hayes [1] for cancellous bone without marrow, showing best fit lines for the Gibson and Ashby, Spriggs, and Rice models.

marrow. The poor behavior of the Spriggs model with the no-marrow data was almost certainly due to the porosity level being higher than that for which the model is considered valid (i.e., porosity lying in the range for which the Rice model is more appropriate).

When the Rice high-porosity modification of the Spriggs model was used, the predictions for the tissue modulus substantially improved. With this model, E_0 was predicted as 8.9 GPa (with marrow), 6.7 GPa (without marrow), and 5.0 GPa (combined), in excellent agreement with the published values for cancellous bone tissue. The Solver also predicted a realistic value for the pore geometry factor, b'. Statistically, the Rice model provided a fit which was good ($p \ll 0.0001$ in all cases), and comparable to the best of the other models. It was also the only model to predict a reduction in stiffness of the bone when the marrow is removed.

The Nielsen model was found to be inappropriate for cancellous bone. In all cases the solution converged to a pore geometry factor, k_N, which was outside the legitimate range.

The Rice high-porosity modification of the Spriggs model appears to be viable for describing and predicting the elastic modulus of cancellous bone. It predicts a realistic value for the tissue modulus, lying within the range of measured values for dense cortical bone. It is also the only model to predict an increase in Young's modulus when marrow is present in the bone.

5. Conclusions

The cellular solid model for open porosity ($E \propto \rho^2$) was found to give a statistically good fit to the data of Carter and Hayes. However, it was found to substantially underpredict the tissue modulus. Better predictions of tissue modulus were found for the closed porosity cellular solid model ($E \propto \rho^3$), but the fit to the data was statistically somewhat poorer. In both cases, the models predicted that bone containing marrow should be less stiff than bone without marrow.

The minimum solid area model of Spriggs was not found to be adequate for modeling highly porous cancellous bone, with substantial overprediction of the tissue modulus of bone from which the marrow had been removed. Similarly, the Nielsen model was found to be inapplicable to cancellous bone.

The Rice modification of the Spriggs model was found to combine a good prediction of tissue modulus with a good statistical fit to the data, and should prove useful for predicting the Young modulus of cancellous bone for biomechanical work.

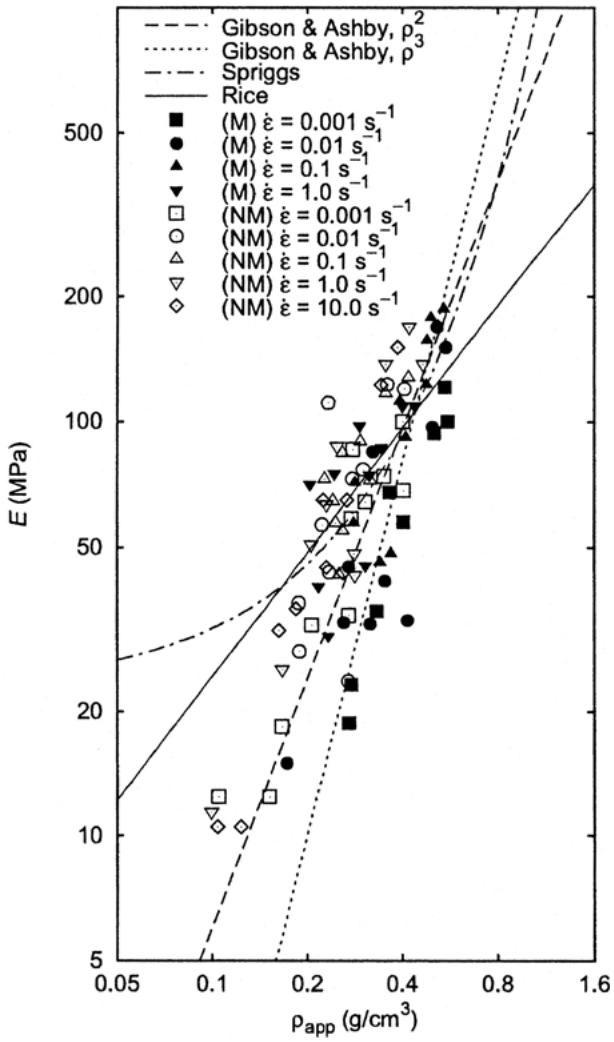

Figure 5 Young's modulus data from Carter and Hayes [1] for cancellous bone with and without marrow, showing best fit lines for the Gibson and Ashby, Spriggs, and Rice models. (M = samples containing marrow; NM = samples containing no marrow.)

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References

1. D. R. CARTER and W. C. HAYES, *J. Bone Joint Surg.* **59-A** (1977) 954.
2. J. D. CURREY, *J. Biomechanics* **21** (1988) 131.
3. R. HODGKINSON and J. D. CURREY, *J. Mater. Sci. Mater. Med.* **3** (1992) 377.
4. M. DALSTRA, R. HUISKES, A. ODGAARD and L. VAN Erning, *J. Biomechanics* **26** (1993) 523.
5. L. J. GIBSON, *ibid.* **18** (1985) 317.
6. K. RAJAN, *J. Mater. Sci. Lett.* **4** (1985) 609.
7. L. J. GIBSON and M. F. ASHBY, *Proc. R. Soc. Lond. A* **382** (1982) 43.
8. J. C. RICE, S. C. COWIN and J. A. BOWMAN, *J. Biomechanics* **21** (1988) 155.
9. R. M. CHRISTENSEN, *J. Mech. Phys. Solids* **34** (1986) 563.
10. B. VAN RIETBERGEN, H. WEINANS, R. HUISKES and A. ODGAARD, *J. Biomechanics* **28** (1995) 69.
11. J. WOLFF, ''Das Gesetz der Transformation der Knochen'' (Hirschwald, Berlin, 1892).
12. J. Y. RHO, R. B. ASHMAN and C. H. TURNER, *J. Biomechanics* **26** (1993) 111.
13. J. Y. RHO, M. E. ROY, T. Y. TSUI and G. M. PHARR, *J. Biomed. Mater. Res.* **45** (1999) 48.
14. J. D. ESHELBY, *Proc. R. Soc. Lond. A* **241** (1957) 376.
15. L. F. NIELSEN, *Mater. Sci. Engng* **52** (1982) 39.
16. L. F. NIELSEN, *J. Am. Ceram. Soc.* **67** (1984) 93.
17. N. RAMAKRISHNAN and V. S. ARUNACHALAM, *ibid.* **76** (1993) 2745.
18. R. M. SPRIGGS, *ibid.* **44** (1961) 628.
19. F. P. KNUDSEN, *ibid.* **45** (1962) 94.
20. R. M. SPRIGGS, L. A. BRISSETTE and T. VASILOS, *ibid.* **45** (1962) 400.
21. R. W. RICE, *ibid.* **58** (1975) 458.
22. R. W. RICE, *ibid.* **59** (1976) 536.
23. R. W. RICE, *ibid.* **76** (1993) 1801.
24. R. W. RICE, *J. Mater. Sci.* **31** (1996) 1509.
25. K. PIEKARSKI, in ''Orthopaedic Mechanics: Procedures and Devices'', edited by D. N. Ghista and R. Roaf (Academic Press, New York, 1978) p. 1.
26. J. K. MACKENZIE, *Proc. Phys. Soc. Lond. B* **63** (1950) 2.
27. R. L. COBLE and W. D. KINGERY, *J. Am. Ceram. Soc.* **39** (1956) 377.
28. R. H. MYERS, ''Classical and Modern Regression with Applications'', 2nd edn (Duxbury Press, Belmont, CA, 1990) p. 444.
29. E. RYSHKEWITCH, *J. Am. Ceram. Soc.* **36** (1953) 65.
30. W. DUCKWORTH, *ibid.* **36** (1953) 68.
31. F. P. KNUDSEN, *ibid.* **42** (1959) 376.
32. J. B. WACHTMAN, ''Mechanical Properties of Ceramics'' (Wiley–Interscience, New York, 1996) p. 410.
33. S. D. BROWN, R. B. BIDDULPH and P. D. WILCOX, *J. Am. Ceram. Soc.* **47** (1964) 320.
34. J. GALANTE, W. ROSTOKER and R. D. RAY, *Cal. Tiss. Res.* **5** (1970) 236.
35. W. MENDENHALL and T. SINCICH, ''Statistics for Engineering and the Sciences'', 4th edn (Prentice-Hall, Upper Saddle River, NJ, 1994) p. 536.
36. P. R. TOWNSEND and R. M. ROSE, *J. Biomechanics* **8** (1975) 199.
37. R. B. ASHMAN and J. Y. RHO, *ibid.* **21** (1988) 177.
38. P. L. MENTE and J. L. LEWIS, *J. Orthop. Res.* **7** (1989) 456.
39. K. CHOI, J. L. KUHN, M. J. CIARELLI and S. A. GOLDSTEIN, *J. Biomechanics* **23** (1990) 1103.

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